Inline Hologram Notes

Dr Phil Birch

January 6, 2016

The object wave is

$$O(x, y) = A(x, y)\exp(i\phi(x, y)) \tag{1}$$

and the 4 reference waves plane waves but with phase shifts and are

$$R_1 = R\exp(0) = R \tag{2}$$
$$R_2 = R\exp(i\pi/2) = iR \tag{3}$$
$$R_3 = R\exp(i\pi) = -R \tag{4}$$
$$R_4 = R\exp(i3\pi/2) = -iR \tag{5}$$

Dropping the (x,y) notation, the four inline holograms are then

$$I_1 = R^2 + A^2 + RA\exp(i\phi) + RA\exp(-i\phi) \tag{6}$$
$$I_2 = R^2 + A^2 - iRA\exp(i\phi) + iRA\exp(-i\phi) \tag{7}$$
$$I_3 = R^2 + A^2 - RA\exp(i\phi) - RA\exp(-i\phi) \tag{8}$$
$$I_4 = R^2 + A^2 - iRA\exp(i\phi) - iRA\exp(-i\phi) \tag{9}$$

These are the images recorded on a CCD. To reconstruct, we illuminate each hologram by its reference beam.

$$R_1I_1 = R^3 + RA^2 + R^2A\exp(i\phi) + R^2A\exp(-i\phi) \tag{10}$$
$$R_2I_2 = iR^3 + iRA^2 + R^2A\exp(i\phi) - R^2A\exp(-i\phi) \tag{11}$$
$$R_3I_3 = -R^3 - RA^2 + R^2A\exp(i\phi) + R^2A\exp(-i\phi) \tag{12}$$
$$R_4I_4 = -iR^3 - iRA^2 + R^2A\exp(i\phi) - R^2A\exp(-i\phi) \tag{13}$$

Add together the four complex fields to get

$$U = 4R^2A\exp(i\phi) \tag{14}$$

which is a virtual image and needs a lens to reconstruct. We can change the order of the reference fields to

$$R_1 I_1 + R_4 I_2 + R_3 I_3 + R_2 I_4 \tag{15}$$

which gives

$$U = 4R^2 A \exp(-i\phi) \tag{16}$$

which results in a real image so we only need to propagate U with Fresnel diffraction to get the reconstruction.